Radar Tutorial Solution - Flt Lt Woodward

These notes aim to document my thought process for the design of the surveilance radar tutorial.

Wavelength for a 5GHz signal.

In[65]:= $\lambda = \frac{3 \times 10^8}{5. \times 10^9}$

Out[65]= 0.06

Height required at the radar antenna for a 45km range with a tgt at 75m.

In[26]:= $\left(\frac{45}{4.1} - \sqrt{75}\right)^2$

Out[26]= 5.36087

Even though the radar being 5.4m up will give a line of sight, the ground will impinge on the Fresnel zone, so we need to do something about this later.

15 degree beam width converted to radians.

In[30]:= $15. / 360 \times 2\,\mathrm{Pi}$

Out[30]= 0.261799

Maximum vertical dimension of the antenna to achieve a 15 degree beam width in elevation. $\frac{\lambda}{\theta}$

In[34]:= 0.06 / 0.262

Out[34]= 0.229008

Calculate the maximum height covered by a 15 degree beam at max range. Tanθ = Opp / Adj.

In[36]:= $45. \times 10^3$ Tan[15 Degree]

In[37]:= convert 12060 metres to feet

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UnitConvert[12 060 m, "Feet"]
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Out[37]= $\frac{5\,025\,000}{127}$ ft

In[38]:= N[$\frac{5\,025\,000}{127}$ ft, 6]

Out[38]= 39 566.9 ft

We assumed a horizontal dimension to the antenna of 3m, so we no compute the azimuth beam width.

In[39]:= 0.06 / 3

Out[39]= 0.02

In[40]:= $\frac{0.02}{2\,\text{Pi}}\,360$

Out[40]= 1.14592

Given both of the beam widths, we can calculate the antenna gain, assuming a rectangle.

In[41]:= $\frac{42\,000.}{15\times 1.15}$

Out[41]= 2434.78

and convert to dBs. (for a sanity check)

In[44]:= `N[10 Log[10, 2434]]`

Out[44]= 33.8632

Next, the probability of detection...

0.98 (from the spec) = $\binom{4}{3} P_D{}^3(1-P_D) + \binom{4}{4} P_D{}^4(1-P_D)^0$

0.98 = $4.\,P_D{}^3(1-P_D) + 1.\,P_D{}^4\,1$

In[46]:= `Binomial[4, 3]`

Out[46]= 4

In[48]:= `Solve`$[4\,x^3 - 3\,x^4 == 0.98,\ x]$

Out[48]= $\{\{x\to -0.331096 - 0.468634\,i\},\ \{x\to -0.331096 + 0.468634\,i\},\ \{x\to 0.93986\},\ \{x\to 1.05566\}\}$

The only relevant solution is 0.9398, or 0.94.

Given this probability of detection for one scan, and an assumed false alarm rate of 10^{-6}, we look up the required SNR from the graph. 13.5 dB. Converted to linear numbers.

In[57]:= $10^{\frac{13.5}{10}}$

Out[57]= 22.3872

Next, Band width, c / 2. ΔR

In[49]:= $\frac{3\times 10^8}{2\times 30.}$

Out[49]= $5.\times 10^6$

This goes together with an uncompressed pulse width.

In[70]:= $\frac{1.}{5\times 10^6}$

Out[70]= $2.\times 10^{-7}$

or 0.2 μSec.

Un-ambiguous range PRF

In[50]:= $\frac{3\times 10^8}{2.\times 45\times 10^3}$

Out[50]= 3333.33

We assumed a RPM of 15, i.e. a 4 sec scan. This seems reasonable, given the furthest a target can progress is 4 x 200 ms^{-1} or 800m and

In[91]:= **45 000 / 800.**

Out[91]= 56.25

this gives 56 seconds to detect it.

Number of pulses in each beam dwell, PRF x beam width / 6. RPM.

In[80]:= $$\frac{3333 \times 1.15}{6 \times 15}$$

Out[80]= 42.5883

We’re now in a position to make our first estimate at the peak power output required.

$$Pt = \frac{R^4 (4\pi)^3 k.\ T.\ B.\ SNR}{G^2 \lambda^2 \sigma\ n\ E_i}$$

Additional assumptions:
Temp is 1000k to allow for the receiver noise and noise figure.
E_i is 0.8 (data sheet suggests between 0.7 and 0.9 for a coherent receiver).

In[62]:= $$\frac{45000^4\ (4\ \mathrm{Pi})^3\ 1.38 \times 10^{-23}\ 1000 \times 5 \times 10^6\ 22.4}{2434^2\ 0.06^2\ 10 \times 42 \times 0.8}$$

Out[62]= 1755.07

This regime has a duty cycle of Pulse width x PRF.

In[72]:= $2. \times 10^{-7}\ 3333$

Out[72]= 0.0006666

This duty cycle is way below what is achievable. i.e. there is much scope for higher PRF and / or longer pulses. However, a longer pulse will require pulse compression to maintain range resolution and a higher PRF will require PRF agility (probably stagger) to maintain the un-ambiguous range. Increasing the pulse width has the added advantage of reducing the peak power by spreading it over a longer time period to maintain the same energy.

This does not take into account rain, or other weather phenomina, or any losses. Put simply, peak power output could easily be double or more, but we have no means of making an assessment with the given data. Clutter, however, will be a problem, so we need to make assessment of the possibility of a MTI mode. We need to choose a PRF so that the blind speed of the radar will be greater than 200 ms^{-1}.

Blind velocities $V_{blind} = \frac{n.\lambda PRF}{2}$ where n ∊ ℤ+. We’re concerned with the first, i.e. n = 1.

In[68]:= $$\frac{0.06 \times 3333}{2}$$

Out[68]= 99.99

so we would need to double our PRF for the max speed to correspond to the MTI first blind speed. i.e. we would need a PRF of 10000Hz to be really clear. The max un-ambiguous range for a PRF of 10KHz is

In[73]:= $$\frac{3. \times 10^8}{2. \times 10000}$$

Out[73]= 15 000.

i.e. 15Km, well below what is needed. If I use a 2 position stagger with a PRF of 10KHz and a PRF with max unambiguous range of 9km, the combination will have a LCM of 45km as required.

In[74]:= $$\frac{3\times 10^{8}}{2.\times 9\times 10^{3}}$$

Out[74]= 16 666.7

If I also alternate them is a pair wise fashion I end up with a 2 element, 2 position stagger. Alternatively, I could use a 2 element 5 position stagger, to increase the pulse density. ie, PRF 1 is 10KHz, PRF 2 is 16.667KHz. and I fire 1,2,1,1,2. This sequece has the total duration of 2/10000 + 3/16667

In[77]:= **2. / 10 000 + 3 / 16 667**

Out[77]= 0.000379996

and the average PRF is

In[79]:= **Mean[{10 000., 16 667, 10 000, 16 667, 16 667}]**

Out[79]= 14 000.2

and this pattern can fit into the beam dwell time ... times.

In[81]:= $$\frac{14\,000\times 1.15}{6\times 15}$$

Out[81]= 178.889

this has the duty cycle of

In[84]:= $2.\times 10^{-7}\ 14\,000$

Out[84]= 0.0028

i.e. approx 0.3%. This can be increased further, so we now consider pulse compression, to maintain the range resolution, but increase the pulse width. If we aim for a 10% duty cycle, I can increase my pulse duration by 10/0.28

In[85]:= **10 / 0.28**

Out[85]= 35.7143

In[87]:= $2\times 10^{-7} * 35.$

Out[87]= $7.\times 10^{-6}$

The pulse is now 7*μ*s, but the band width is still the same, so I compute γ, the rate of linear chirp required. So if we have a compression ratio of 35, the change in frequence required over the new 7*μ*s pulse is

In[89]:= $$\frac{35}{7.\times 10^{-6}}$$

Out[89]= $5.\times 10^{6}$

i.e. a linear chirp of 5MHz. Because of the increase in noise in the receiver due to the longer pulse, the this has no effect of the detection. However, we can re-compute the range equation for the revised figures.

In[90]:= $$\frac{45\,000^{4}\,(4\,\mathrm{Pi})^{3}\ 1.38\times 10^{-23}\ 1000\times 5\times 10^{6}\ 22.4}{2434^{2}\ 0.06^{2}\ 10\times 179\times 0.8}$$

Out[90]= 411.804

So we have a peak power requirement of 411 watts (same caveates as before - probably more like 1kW), and with the new PRF pattern and Chirp, a 10% duty cycle.